APM3713 Assignment 3

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Question 1

(a)

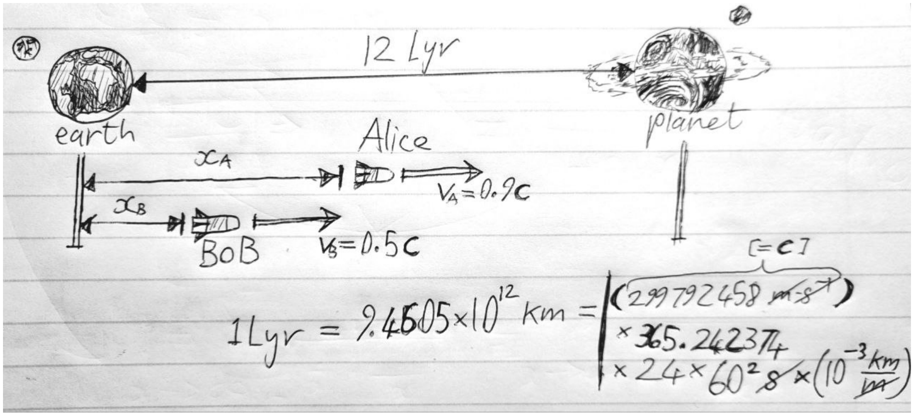

Figure ⟨I⟩: Alice and Bob visual with info

Give the velocity of Alice and Bob, is $V_A = 0.9c$ and $V_B = 0.5c$ respectivly. We also know that one light-year (ly), is calculated as followes using, $c \stackrel{\text{def}}{\equiv} 299792458\ \text{m}{\cdot}\text{s}^{-1}$.

$$1\text{ ly} = c \cdot (365.242374\ \text{day}\ \times 24\ \text{h/day}\ \times 60^2 s \cdot h^{-1})(10^{-3}\tfrac{km}{m})$$
$$1\text{ ly} = 9.4605 \times 10^{12}\text{km},$$

and so 12 ly $= 12 \times (9.4605 \times 10^{12}$ km).

> Lemma
>
> $\Delta t = t_2 - t_1,\ \ \Delta x = 0,\ \ \Delta x' = \mathcal{L}_P$
> $L = d_2 - d_1$, where $Vt_2 = d_1$ and $Vt_1 = d_2$.
> $\because\ \ \Delta x' = \gamma(\Delta x - c\Delta t \cdot \frac{V}{c})$
> $\mathcal{L}_P = \gamma(0 + [-\Delta t V = Vt_1 - Vt_2]) = \gamma(d_2 - d_1),$
> $\therefore\ \ \mathcal{L}_P = \gamma L$, or as...

$$L = \frac{\mathcal{L}_P}{\gamma},\ \text{length contraction equation.}$$

For Alice

$$L_{\text{Alice}} = \frac{\mathcal{L}_P}{\gamma} = 12\text{ ly}\sqrt{1 - V^2/c^2} = 12 \times 9.4605 \times 10^{12}\ \text{km}\cdot\sqrt{1 - 0.9^2}$$

$$\boxed{L_{\text{Alice}} = 4.9485 \times 10^{13}\ \text{km}}$$

(b)

For Bob, given that $V_{_B} \cdot \Delta t_{\text{Bob}} = 12 \cdot c \text{ yr} = 12 \text{ ly}$. Using the time dilation equation,

$$\Delta t_{\text{Bob}} = \gamma \Delta\tau \qquad (\Delta\tau \equiv \Delta t'_{\text{Bob}}) \ ,$$

along the start and end (event) points of Bob's journy in $S'_{(\text{Bob})}$.

$$V_{_B} \cdot \Delta t_{\text{Bob}} = 12 \cdot c \text{ yr} = V_{_B} \cdot \gamma \Delta\tau \ \ = 0.5c \times \frac{\Delta\tau}{\sqrt{1-(0.5c)^2/c^2}}$$

$$12 \cdot c \text{ yr} = 0.5c \times \frac{\Delta\tau}{\sqrt{1-(0.5c)^2/c^2}}$$

$$\Delta\tau = \left(\frac{12}{0.5}\right) \text{ yr } \times\sqrt{0.75}, \qquad \text{where unit yr = year.}$$

$$\Delta\tau = 20.78 \text{ yr}$$

SI basis unit for dimension time[T], is the unit second, s. Finally converting from yr to s (1 yr$= 315.5694 \times 10^5$ s) gives the duration in Bob's frame as

$$\boxed{\Delta\tau = 6.558 \times 10^8 \text{ s}}$$

(c)

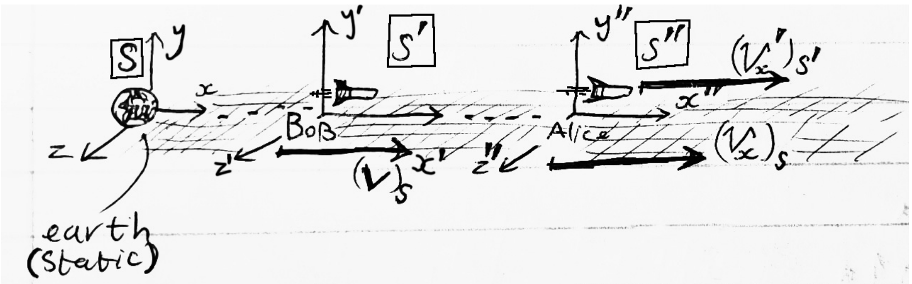

Figure ⟨II⟩: $S_{\{\text{static}\}}, S', S''$; the frames for earth, Bob, and Alice. Here the notation $B \lessdot A$, has the meaning "*Alice relative to Bob*/(*observed by Bob*)".

$$v_{_{B\lessdot A}} = \frac{v_{_A} - V_{_B}}{1 - \frac{V_B}{c^2} v_{_A}} \quad v_{_{B\lessdot A}} = \frac{0.9c - 0.5c}{(1 - \frac{0.5\cancel{c}}{\cancel{c^2}} 0.9\cancel{c})} = \frac{0.9 - 0.5}{1 - 0.5 \times 0.9} c = \frac{8}{11} c$$

$$\therefore v_{_{B\lessdot A}} = 0.\overline{72} \ c$$

$$\boxed{v_{_{B\lessdot A}} \approx 0.7273c}$$

(d)

$$f_r = \sqrt{\frac{c-v}{c+v}} \cdot f_s, \quad \begin{cases} \text{if away:} & v > 0 \\ \text{if towards:} & v < 0 \end{cases}$$

$$f_r = \sqrt{\tfrac{c-v}{c+v}} \cdot f_s = \sqrt{\tfrac{c-c0.5}{c+c0.5}} \cdot 400 \text{ Hz} = \sqrt{\tfrac{\cancel{c}(0.5)}{\cancel{c}(1.5)}} \cdot 400 \text{ Hz} = 230.94 \text{ Hz}$$

$$\boxed{f_r = 230.94 \text{ Hz}}$$

unfortunatly I ran out of time to do futher LATEX formating, so the rest of the Assignment is hand writen, but the content is the same.

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Q2

inertial ref. frames S and S' in std conf.

- $V = 0.8c$, for S' rel S, in $+\hat{x}$ dir.
- O' in S' stat ⊲ events A and B,
  at same place $x'_A = x'_B = 250$ m,
  (event A) $t'_A = 1\times10^{-8}$ s, (event B) $t'_B = 7\times10^{-8}$ s.

(a) event B time rel $S \Rightarrow t_B$

$$\because \Delta x' = x'_B - x'_A = 0 \text{ in } S'$$

$$\Delta t' = t'_B - t'_A$$

$$\Delta t = \gamma\left(\Delta t' + \cancelto{0}{\frac{V\Delta x'}{c^2}}\right) = \gamma \Delta t' \to \gamma \Delta T$$

(let $\Delta T = \Delta t'$)

$$\therefore \Delta t = \gamma \Delta T = \gamma \Delta t'$$

$$\therefore \boxed{t_B = \gamma\left(t'_B + \frac{V x'_B}{c^2}\right)}$$

$$\therefore t_B = \frac{1}{\sqrt{1-\frac{V^2}{c^2}}}\left(7\times10^{-8}\,\text{s} + \frac{0.8\cancel{c}\times 250\,\text{m}}{c^{\cancel{2}}}\right)$$

$$t_B = \frac{1}{\sqrt{1-0.8^2\frac{\cancel{c^2}}{\cancel{c^2}}}} \times \left(7\times10^{-8}\,\text{s} + \frac{0.8\times250\,\cancel{\text{m}}\,(\text{s})}{299792458\,\cancel{\text{m}}\,\cancel{\text{s}^{-1}}}\right)$$

$$= 1.228547\times10^{-6}\,\text{s}$$

$$\boxed{t_B = 1.23\times10^{-6}\,\text{s}}$$

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8:07 Pm ~~[illegible]~~

(b) $(\Delta S)^2 \equiv (\Delta S')^2$ invariant

as stated $\Delta y, \Delta z, \Delta y', \Delta z' = 0$

$(\Delta S')^2 = (c\Delta t')^2 - (\Delta x')^2 - (\Delta y')^2 - (\Delta z')^2$

same for unprimed t, x, ... etc

in S' frame $\Delta x' = 0$

$\therefore$

$$(\Delta S')^2 = (c\Delta t')^2 - 0 = (c\Delta T)^2$$

$$\therefore (\Delta S)^2 = c^2 \Delta T^2$$

$$(\Delta S)^2 = (299792458\ m\,s^{-1})^2 \times (t_B' - t_A')^2$$

$$= (299792458\ m\,s^{-1})^2 \times ((7-1)\times 10^{-8}\,s)^2$$

$$\boxed{(\Delta S)^2 = 323.55\ m^2}$$

(C)

if $(\Delta S)^2 > 0$: time-like sep $\exists\ \Delta l = 0, \Delta t \neq 0$ frame.

if $(\Delta S)^2 = 0$: light-like sep ; ~~[illegible]~~ $\Delta l = c\Delta t$, light linked signal.

if $(\Delta S)^2 < 0$: space-like sep $\exists\ \Delta l \neq 0, \Delta t = 0$ frame.

$\Delta S^2 = 323.55\ m^2 > 0$: time like seperated

S is correct, Event A may have caused Event B as events lie inside the light cone (even on light cone is possible via light signal) for massive objects Event A and Event B lie inside each others light cones and share a common futur / past resp.

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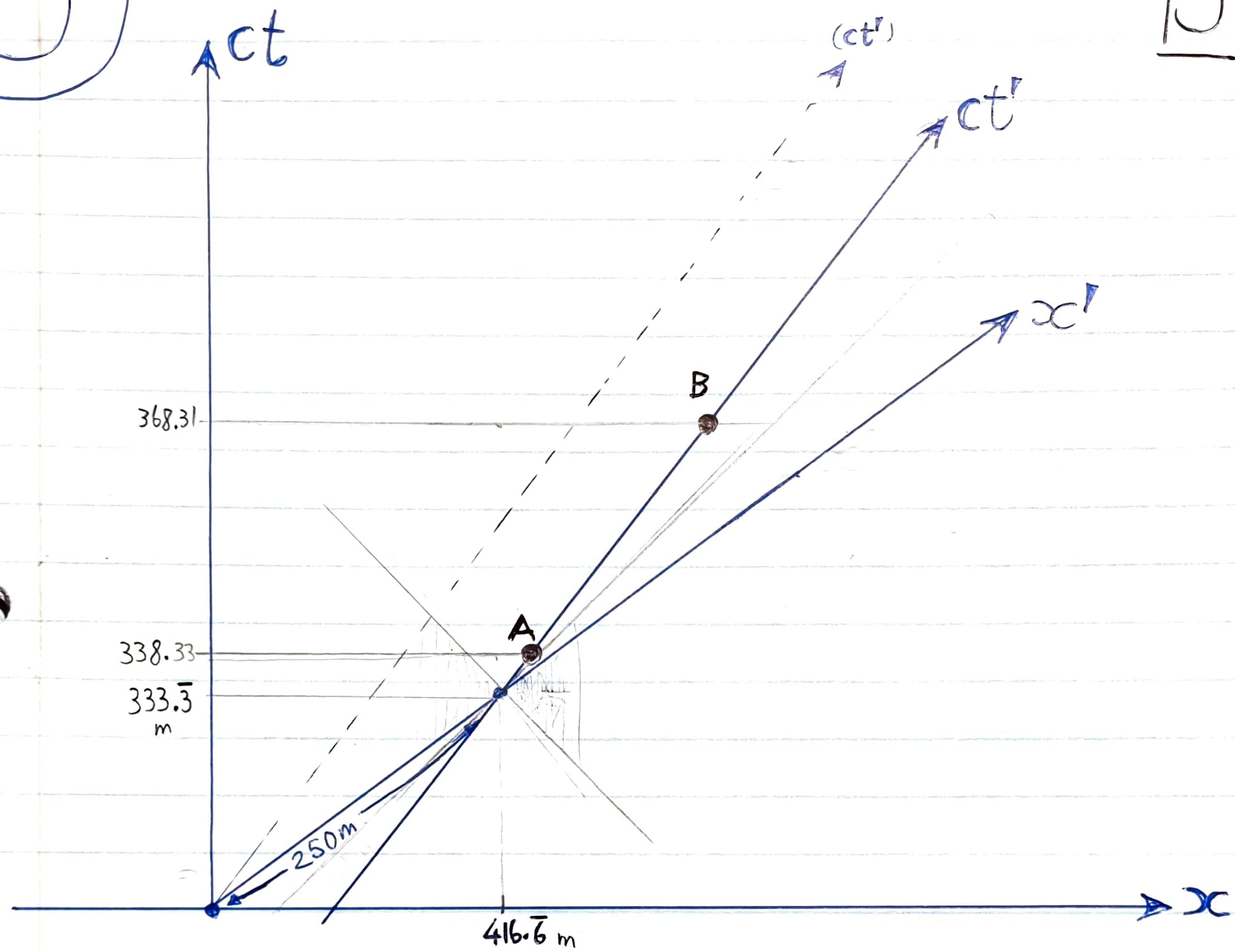
S



(ct'-axis): $ct = x\frac{c}{V}$, $ct = 1.25x$ [ct'] .

(x'-axis): $ct = x\frac{V}{c}$, $ct = 0.8x$ [x'] .

$$c\Delta t = \gamma\left(c\Delta t' + \Delta x'\frac{V}{c}\right) = \gamma c\Delta t' \quad (\Delta x' = 0)$$

$$\boxed{c\Delta t = \gamma c\Delta t'}$$

$$c\Delta t = \gamma c(t'_B - t'_A)$$

$$\boxed{ct = \gamma\left(ct' + x'\frac{V}{c}\right)}$$

$\therefore$

$c\cdot\Delta_0 t_A = 338.33 - 333.33 = 5$ m

$c\cdot\Delta_0 t_B = 368.31 - 333.33 \approx 35$ m

$c\cdot\Delta t = \gamma c(t'_B - t'_A) =$

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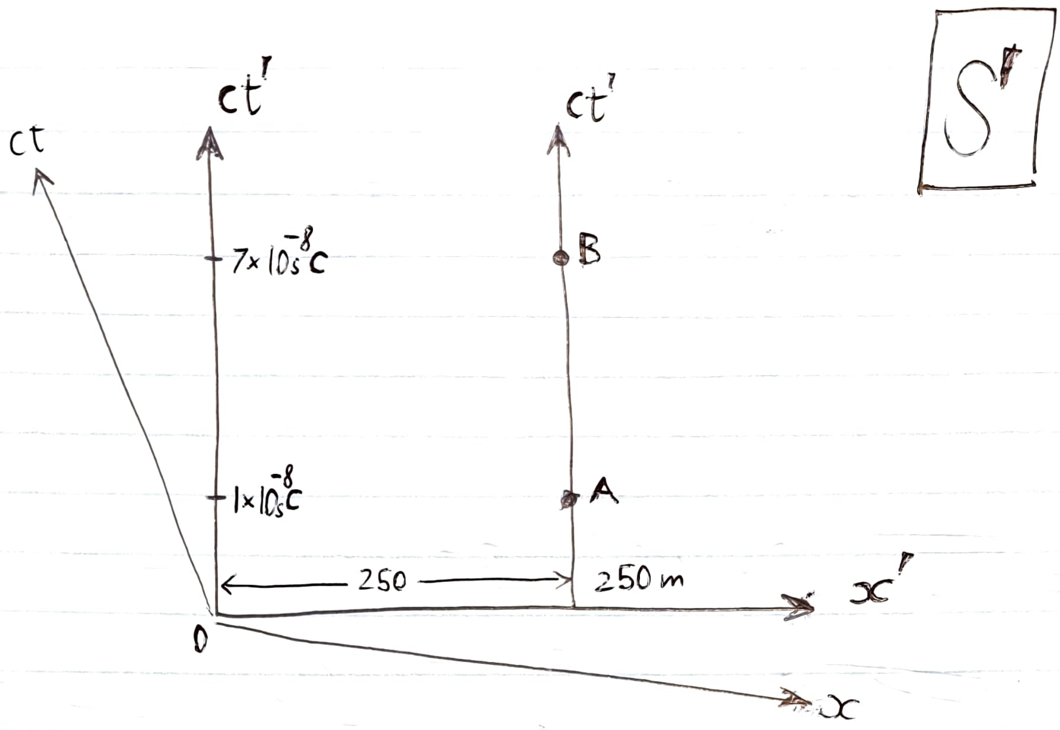

Q3

(a) $2\,GeV = E$, general antiproton energy (total E)

$0.938\,GeV/c^2 = m$, rest mass .

$$E = \gamma m c^2$$

$$E = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} mc^2 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} 0.938\,GeV/\cancel{c^2} \times \cancel{c^2}$$

$$2\,\cancel{GeV} = 0.938\,\cancel{GeV} \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{0.938}{2} = \sqrt{1-\frac{v^2}{c^2}}, \quad \sqrt{1-\frac{0.938^2}{2^2}} = \frac{v}{c}$$, and so

$$\therefore \; V = c\sqrt{1-\left(\frac{0.938}{2}\right)^2} = 299792458\,m\cdot s^{-1}\sqrt{1-\frac{0.938^2}{4}}$$

$V = 15886568.94\ km\cdot min^{-1}$, or as

$\boxed{V = 264\,776\,149\ m\cdot s^{-1}}$ speed.

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$\boxed{v = 0.8832\,c}$

$\boxed{(b)}$ $p = \gamma m v$

$m = 0.938\ \frac{GeV}{c^2}$

$1\ GeV = 1 \times 10^9\ eV = 1.602176634 \times 10^{-19+9}\ J$

$1\ J = 1\ kg \cdot m^2 \cdot s^{-2}$

$c^2 = (299792458\ m \cdot s^{-1})^2$

$$1 = \left(\frac{1.602176634 \times 10^{-10}}{(299792458)^2} \times \frac{kg \cdot c^2}{GeV} \right)$$

$$= 1.782662 \times 10^{-27}\ kg/(GeV \cdot c^{-2})$$

$m = 0.938\ GeV \cdot c^{-2} \times [1.782662 \times 10^{-27}\ kg/(GeV \cdot c^{-2})]$

$m = 1.672137\ \cancel{kg} \times 10^{-27}\ kg$

$$p = \frac{1}{\sqrt{1 - \left(\frac{264776149}{299792458}\right)^2}} \times 1.672137 \times 10^{-27}\ kg \times (264776149\ m \cdot s^{-1})$$

$\boxed{p = 9.4401 \times 10^{-19}\ kg \cdot m \cdot s^{-1}}$

$\boxed{p = 1.7664\ GeV/c}$

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(c) $KE = (\gamma - 1) mc^2$

$= \left(\frac{1}{\sqrt{1-(0.8832)^2}} - 1\right) 0.938\ GeV/c^2 \times c^2$

$KE = 1.0620\ GeV$

$\xrightarrow[\text{fig (3)}]{\text{sig}} \boxed{KE \simeq 1.06\ GeV}$

(d)

$[P^\mu] = \left(\frac{E}{c}, \gamma m v_x, \gamma m v_y, \gamma m v_z\right)$

$= \left(\frac{E}{c}, \vec{p}\right) \equiv (P^0, P^1, P^2, P^3)$

$= m\left(c\frac{dt}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau}\right)$

$\therefore [P^\mu] = m\left[\frac{dx^\mu}{d\tau}\right], \quad P^\mu = m\sum_{\nu=0}^{3} \gamma \frac{d}{dt} x^\nu.$

$\Rightarrow \boxed{[P^\mu] = (2\ GeV/c, 1.7664\ GeV/c, 0, 0)}$

(e)

$$[P'^\nu] = \underbrace{\begin{bmatrix} \gamma(v) & -\gamma(v)\frac{v}{c} & 0 & 0 \\ -\gamma(v)\frac{v}{c} & \gamma(v) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{[\Lambda^\nu{}_\mu]} \underbrace{\begin{bmatrix} 2 \\ 1.7664 \\ 0 \\ 0 \end{bmatrix}}_{[P^\mu]} GeV/c$$

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$V = -0.4c$

$$[P'^{\nu}] = (P'^{0}, P'^{1}, P'^{2}, P'^{3})$$

$$P'^{\nu} = \sum_{\mu=0}^{3} \Lambda^{\nu}{}_{\mu} P^{\mu}$$

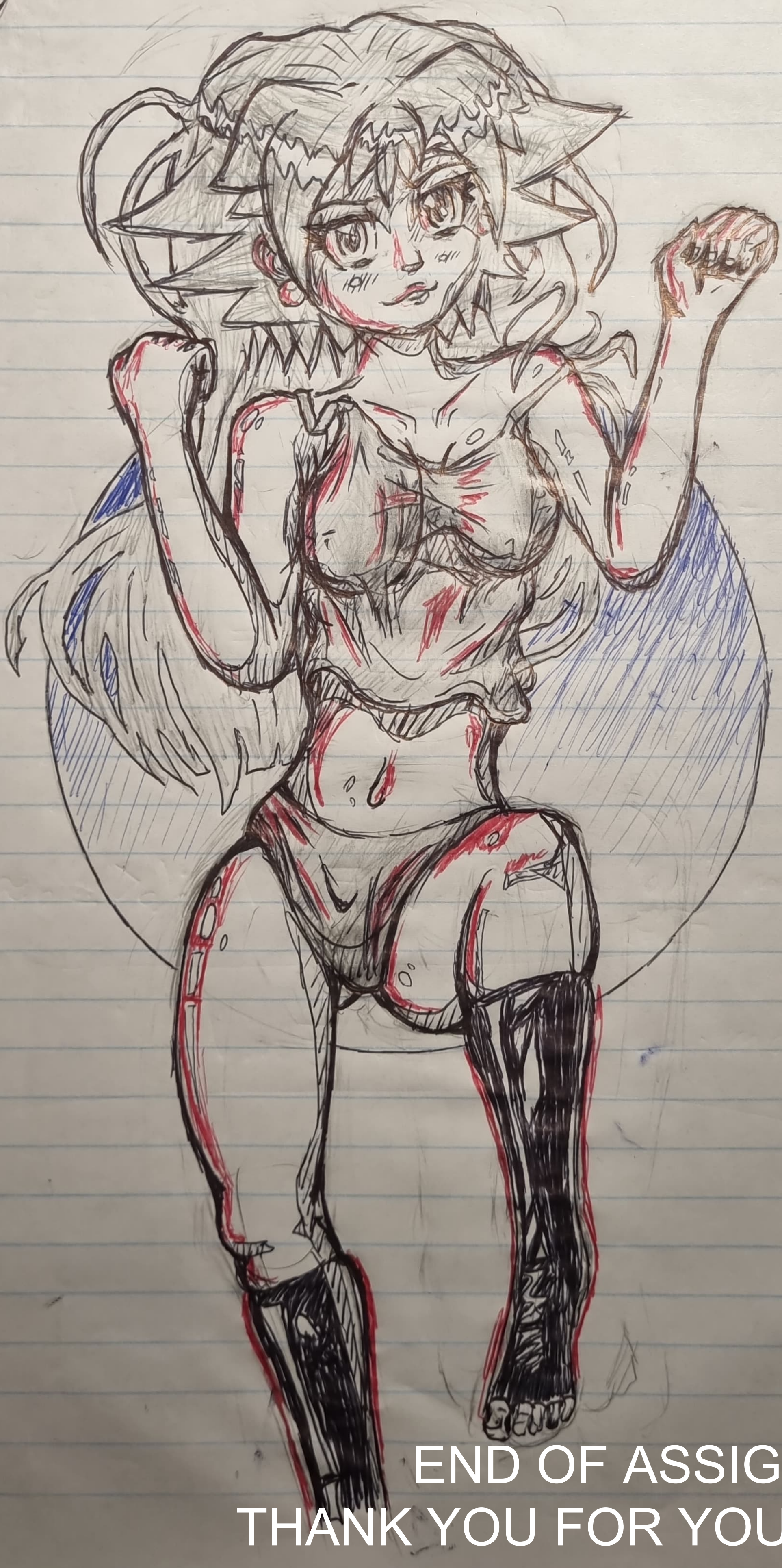
$$P'^{0} = (2\gamma - \gamma \tfrac{V}{c} 1.7664)\ \text{GeV/c}$$
$$P'^{1} = (-2\gamma \tfrac{V}{c} + \gamma 1.7664)\ \text{GeV/c}$$
$$P'^{2} = 0$$
$$P'^{3} = 0$$

$$P'^{0} = \gamma\, \text{GeV/c} \left(2 - \frac{-0.4\cancel{c}}{\cancel{c}} 1.7664\right)$$
$$= \frac{1}{\sqrt{1 - \frac{0.4^2 \cancel{c^2}}{\cancel{c^2}}}} (2 + 0.4 \times 1.7664)\ \text{GeV/c}$$
$$= 2.9531\ \text{GeV/c}$$

$$P'^{1} = \gamma\left(+2\tfrac{V}{c} + 1.7664\right) \text{GeV/c}$$
$$= \frac{1}{\sqrt{1-0.4^2}} (+2 \times 0.4 + 1.7664)\ \text{GeV/c}$$
$$\cancel{= 1.0544\ \text{GeV/c}} = 2.8002\ \text{GeV/c}$$

$$\therefore \boxed{[P'^{\nu}] = \left(\underset{\text{GeV/c}}{2.9531}, \underset{\text{GeV/c}}{2.8022}, 0, 0\right)}$$

END



END OF ASSIGNMENT
THANK YOU FOR YOUR TIME